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Published in:

Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, CDC/CCC 2009

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Document Version

Publisher's PDF, also known as Version of record

Publication date:

2009

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Puerto-Flores, D. D., Ortega, R., & Scherpen, J. M. A. (2009). Power factor compensation with lossless linear filters is equivalent to (weighted) power equalization and a new cyclo-dissipativity characterization. In *Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, CDC/CCC 2009* (pp. 6569-6574). University of Groningen, Research Institute of Technology and Management.

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Power factor compensation with lossless linear filters is equivalent to (weighted) power equalization and a new cyclo-dissipativity characterization

Dunstano del Puerto-Flores, Romeo Ortega, and Jacquélien M. A. Scherpen

Abstract—Recently, it has been established that the problem of power factor compensation for nonlinear loads with non-sinusoidal source voltage can be recast in terms of the property of cyclo-dissipativity. Using this framework the classical capacitor and inductor compensators can be interpreted in terms of energy equalization. Unfortunately, the supply rate is a function of the load, which is usually unknown, styming the applicability of this result for compensator synthesis. The purpose of this brief note is to extend this approach in three directions. First, power factor compensation is shown to be equivalent to a new cyclo-dissipativity condition, whose supply rate is now function of the compensator. Second, we consider general lossless linear filters as compensators and show that the power factor is improved if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Finally, we exhibit the gap between the ideal compensator, *e.g.*, the one that achieves unitary power factor, and the aforementioned equalization condition. This result naturally leads to the formulation of a problem of optimization of the compensator topology and its parameters.

I. INTRODUCTION

Optimizing energy transfer from an alternating current (ac) source to a load is a classical problem in electrical engineering. The power factor (PF), defined as the ratio between the real or the active power (average of the instantaneous power) and the apparent power (the product of rms values of the voltage and current), captures the energy-transmission efficiency for a given load. The standard approach to improve the power factor is to place a lossless compensator between the source and the load.

If the load is scalar linear time-invariant (LTI) and the generator is ideal—that is, with negligible impedance and fixed sinusoidal voltage—it is well known that the optimal compensator minimizes the phase shift between the source voltage and current waveforms [1]. The task of designing compensators that aim at improving PF for nonlinear time-varying loads operating in non-sinusoidal regimes is, on the other hand, far from clear.

The effectiveness of capacitive compensation in systems with non-sinusoidal voltages and currents has been widely studied, see *e.g.* [2] and [3]. In [4] it has been shown that capacitive compensation may not be effective for non-sinusoidal voltages. Therefore, a more complex compensator

is required in such situations. Furthermore, most of the approaches used to improve PF are based on *ad-hoc* definitions of reactive power, [3], and a lack of consensus on these definitions produces misunderstanding of power phenomena in circuits with non-sinusoidal voltages and currents.

Recently, in [5] a new framework for analysis and design of (possibly nonlinear) PF compensators for electrical systems operating in non-sinusoidal (but periodic) regimes with nonlinear time-varying loads was presented. This framework proceeds from the aforementioned, universally accepted, definition of PF and does not rely on any axiomatic definition of reactive power. It is shown that PF is improved if and only if the compensated system satisfies a certain cyclo-dissipativity property, [6]. This result has been applied in [7] to analyze passive compensation of a classical half-bridge controlled rectifier with non-sinusoidal source voltage. Unfortunately, the supply rate in [5] depends explicitly on the load, which is typically unknown. Hence, the result cannot be used for compensator synthesis. One contribution of our work is the proof that PF improvement can also be characterized in terms of a new cyclo-dissipativity property where the supply rate is independent of the load and is solely determined by the compensator.

In [5] the case of LTI capacitive or inductive compensation are studied, showing that PF improvement is equivalent to energy equalization. In this work, we extend this result to consider arbitrary LTI lossless filters, and prove that for general lossless LTI filters the PF is reduced if and only if a certain equalization condition between the weighted powers of inductors and capacitors of the load is ensured. Moreover, we exhibit the gap between the ideal compensator, *e.g.*, the one that achieves unitary PF, and the aforementioned equalization condition. The results reported in this paper pave the road for compensator design applications.

II. A NEW CYCLO-DISSIPATIVITY CHARACTERIZATION OF PF COMPENSATION

A. Framework

We consider the energy transfer from an n -phase ac generator to a load, see Figure 1. All signals are assumed to be periodic and have finite power, that is, they belong to

$$\mathcal{L}_2^n = \left\{ x : [0, T) \rightarrow \mathbb{R}^n : \|x\|^2 := \frac{1}{T} \int_0^T |x(\tau)|^2 d\tau < \infty \right\}$$

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where $\|\cdot\|$ is the Euclidean norm. We also define the inner product in \mathcal{L}_2^n as

$$\langle x, y \rangle := \frac{1}{T} \int_0^T x^\top(t) y(t) dt.$$

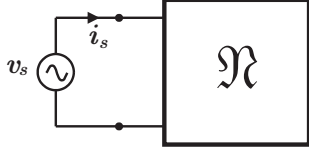


Fig. 1: Circuit schematic of an n -phase ac ideal generator connected to a (possibly nonlinear and time varying) load.

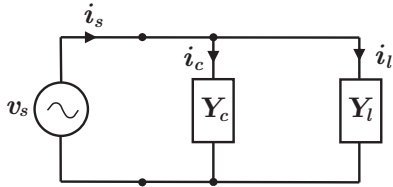


Fig. 2: Circuit schematic of shunt PF compensation configuration.

The universally accepted definition of PF is given as [1]:
Definition 1: The PF of the source is defined by

$$PF := \frac{P}{S}, \quad (1)$$

where

$$P := \langle v_s, i_s \rangle, \quad (2)$$

is the active (real) power,¹ and $S := \|v_s\| \|i_s\|$ is the apparent power.

From (1) and the Cauchy–Schwartz inequality, it follows that $P \leq S$. Hence $PF \in [-1, 1]$ is a dimensionless measure of the energy-transmission efficiency. Cauchy–Schwartz also tells us that a necessary and sufficient condition for the apparent power to equal the active power is that v_s and i_s are collinear. If this is not the case, $P < S$ and compensation schemes are introduced to maximize the PF.

B. The PF compensation problem

The PF compensation configuration considered in the paper is depicted in Figure 2, where $Y_c, Y_\ell : \mathcal{L}_2^n \rightarrow \mathcal{L}_2^n$ are the admittance operators of the compensator and the load, respectively. That is,

$$i_c = Y_c v_s \quad i_\ell = Y_\ell v_s$$

where $i_c, i_\ell \in \mathcal{L}_2^n$, are the compensator and load currents, respectively. In the simplest LTI case the operators Y_c, Y_ℓ can be described by their admittance transfer matrices, which we denote by $\hat{Y}_c(s), \hat{Y}_\ell(s) \in \mathbb{R}^{n \times n}(s)$, respectively.

¹Also called average power [8].

The uncompensated PF, that is, the value of PF when $Y_c = 0$, is clearly given by

$$PF_u := \frac{\langle v_s, i_s \rangle}{\|v_s\| \|i_s\|}. \quad (3)$$

We make the following fundamental assumption.²

Assumption 2: The source is ideal, in the sense that v_s remains unchanged for all loads Y_ℓ .

Following standard practice, we consider only lossless compensators, that is,

$$\langle Y_c v_s, v_s \rangle = 0, \quad \forall v_s \in \mathcal{L}_2^n. \quad (4)$$

We recall that, if Y_c is LTI, this is equivalent to

$$\mathbb{R}_e\{\hat{Y}_c(j\omega)\} = 0. \quad (5)$$

C. PF compensation and cyclo-dissipativity

Definition 3: Given a mapping $w : \mathcal{L}_2^n \times \mathcal{L}_2^n \rightarrow \mathbb{R}$. The n -port system of Figure 1 is cyclo-dissipative with respect to the supply rate $w(v_s, i_s)$ if and only if

$$\int_0^T w(v_s(t), i_s(t)) dt > 0. \quad (6)$$

for all $(v_s, i_s) \in \mathcal{L}_2^n \times \mathcal{L}_2^n$.

To place our results in context, and make the paper self-contained, we recall the following results from [5].

Proposition 4: Consider the system of Figure 2 with fixed Y_ℓ . The compensator Y_c improves the PF if and only if the system is cyclo-dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_\ell v_s + i_s)^\top (Y_\ell v_s - i_s). \quad (7)$$

Proof: From Kirchhoff's current law $i_s = i_c + i_\ell$, the relation $i_c = Y_c v_s$, and the lossless condition (4), it follows that $\langle v_s, i_s \rangle = \langle v_s, i_\ell \rangle$. Consequently, (1) becomes

$$PF = \frac{\langle v_s, i_\ell \rangle}{\|v_s\| \|i_s\|}. \quad (8)$$

Comparing the equation above with (3) we conclude that $PF > PF_u$ if and only if

$$\|i_s\|^2 < \|i_\ell\|^2 = \|Y_\ell v_s\|^2, \quad (9)$$

where we used $i_\ell = Y_\ell v_s$ for the right hand side identity. Finally, note that (6) with (7) is equivalent to (9), which yields the desired result. ■

Corollary 5: Consider the system of Figure 2. Then Y_c improves the PF for a given Y_ℓ if and only if Y_c satisfies

$$2\langle Y_\ell v_s, Y_c v_s \rangle + \|Y_c v_s\|^2 < 0, \quad \forall v_s \in \mathcal{L}_2^n. \quad (10)$$

Dually, given Y_c , the PF is improved for all Y_ℓ that satisfy (10).

Proof: Substituting $i_s = (Y_\ell + Y_c)v_s$ in (9) yields (10). ■

²Under Assumption 2, the apparent power S is the highest average power delivered to the load among all loads that have the same rms current $\|i_s\|$.

D. A new cyclo-dissipativity condition for PF compensation

Proposition 6: Consider the system of Figure 2 with fixed Y_c . The PF is improved for all Y_ℓ such that the system is cyclo-dissipative with respect to the supply rate

$$w(v_s, i_s) := (Y_c v_s)^2 - 2i_s^\top Y_c v_s. \quad (11)$$

Proof: We have shown above that $PF > PF_u$ if and only if $\|i_s\|^2 < \|i_\ell\|^2$. Using the fact that $i_s = i_c + i_\ell$, the latter inequality can be written as

$$\|i_c + i_\ell\|^2 < \|i_\ell\|^2, \quad (12)$$

which is equivalent to

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0. \quad (13)$$

Substituting $i_\ell = i_s - i_c$ in (13) yields

$$\|i_c\|^2 - 2\langle i_c, i_s \rangle > 0. \quad (14)$$

The proof is completed replacing $i_c = Y_c v_s$. ■

The supply rate (7) depends on Y_ℓ that is usually unknown. Hence, the result of Proposition 4 can only be used for analysis of a given known load—as done in [7] for a TRIAC controlled rectifier. On the other hand, the supply rate (11) depends on Y_c , that is to be designed. Current research is under way to exploit this new cyclo-dissipativity property to synthesize PF compensators.

III. WEIGHTED POWER EQUALIZATION AND PF COMPENSATION FOR RLC LOADS

In this section we extend Proposition 5 in [5], where the PF compensators are assumed to be capacitors or inductors, to general lossless LTI filters. Similarly to [5], we assume that the load is a nonlinear RLC circuit consisting of lumped dynamic elements (n_L inductors, n_C capacitors) and static elements (n_R resistors). Capacitors and inductors are defined by the physical laws and constitutive relations [8]:

$$i_C = \dot{q}_C, \quad v_C = \nabla H_C(q_C), \quad (15)$$

$$v_L = \dot{\phi}_L, \quad i_L = \nabla H_L(\phi_L), \quad (16)$$

respectively, where $i_C, v_C, q_C \in \mathbb{R}^{n_C}$ are the capacitors currents, voltages and charges, and $i_L, v_L, \phi_L \in \mathbb{R}^{n_L}$ are the inductors currents, voltages and flux-linkages, $H_L : \mathbb{R}^{n_L} \rightarrow \mathbb{R}$ is the magnetic energy stored in the inductors, $H_C : \mathbb{R}^{n_C} \rightarrow \mathbb{R}$ is the electric energy stored in the capacitors, and ∇ is the gradient operator. We assume that the energy functions are twice differentiable. Resistors are assumed to be linear, that is, $v_R = Ri_R$, with $R \in \mathbb{R}^{n_R \times n_R}$ a diagonal, positive definite matrix. For linear capacitors and inductors

$$H_C(q_C) = \frac{1}{2} q_C^\top C^{-1} q_C, \quad H_L(\phi_L) = \frac{1}{2} \phi_L^\top L^{-1} \phi_L,$$

respectively, with $L \in \mathbb{R}^{n_L \times n_L}$, $C \in \mathbb{R}^{n_C \times n_C}$. To avoid cluttering the notation we assume L, C are diagonal matrices.

Recalling the definition of real power (2) we introduce the following.

Definition 7: Given a compensator admittance Y_c the weighted (real) power of a single-phase circuit with port variables $(v, i) \in \mathcal{L}_2 \times \mathcal{L}_2$ is given by

$$P^w := \langle Y_c v, i \rangle. \quad (17)$$

If Y_c is LTI

$$P^w = \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}[k] \hat{I}^*[k] \quad (18)$$

where $\hat{V}[k], \hat{I}[k]$ are the k -th spectral lines of v and i , respectively, and $\hat{Y}_c[k] := \hat{Y}_c(k\omega_0)$, with $\omega_0 := \frac{2\pi}{T}$. That is, P^w is the sum of the power components of the circuit modulated by the frequency response of Y_c —hence the use of the “weighted” qualifier.³

Proposition 8: Consider the system of Figure 2 with $n = 1$,⁴ a nonlinear RLC load, with linear resistors, and a fixed LTI lossless compensator with admittance transfer function $\hat{Y}_c(s)$.

i) PF is improved if and only if

$$\frac{1}{2} V_s^w + \sum_{q=1}^{n_L} P_{L_q}^w + \sum_{q=1}^{n_C} P_{C_q}^w < 0 \quad (19)$$

where V_s^w is the rms value of the filtered voltage source, that is,

$$V_s^w := \|Y_c v_s\|^2 = \sum_{k=1}^{\infty} |\hat{Y}_c(k) \hat{V}_s(k)|^2$$

and

$$P_{C_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{C_q}[k] \hat{I}_{C_q}^*[k]$$

$$P_{L_q}^w := \sum_{k=-\infty}^{\infty} \hat{Y}_c[k] \hat{V}_{L_q}[k] \hat{I}_{L_q}^*[k],$$

are the weighted powers of the q -th inductor and capacitor, respectively

ii) Condition (19) may be equivalently expressed as

$$\left\langle \left(\frac{1}{p} Y_c \right) v_L, \nabla^2 H_L v_L \right\rangle - \left\langle i_C, \left(\frac{1}{p} Y_c \right) \nabla^2 H_C i_C \right\rangle > \frac{1}{2} V_s^w \quad (20)$$

where $p := \frac{d}{dt}$.

iii) If the inductors and capacitors are linear their weighted powers become

$$P_{C_q}^w := 2\omega_0 \sum_{k=1}^{\infty} \left\{ k \mathcal{I}_m \{ \hat{Y}_c[k] \} \sum_{q=1}^{n_C} C_q |\hat{V}_{C_q}[k]|^2 \right\}$$

$$P_{L_q}^w := -2\omega_0 \sum_{k=1}^{\infty} \left\{ k \mathcal{I}_m \{ \hat{Y}_c[k] \} \sum_{q=1}^{n_L} L_q |\hat{I}_{L_q}[k]|^2 \right\}. \quad (21)$$

³Since the spectral lines of real signals satisfy $\hat{F}[-k] = \hat{F}^*[k]$, the weighted power is a real number.

⁴This condition is imposed, without loss of generality, to simplify the presentation of the result.

Proof: Corollary 5 shows that the PF is improved if and only if (10) holds, which may be equivalently expressed as

$$\|Y_c v_s\|^2 + 2\langle Y_c v_s, i_\ell \rangle < 0.$$

Applying Tellegen's theorem to the RLC load one gets

$$i_\ell^\top Y_c v_s = i_R^\top Y_c v_R + i_L^\top Y_c v_L + i_C^\top Y_c v_C,$$

which upon integration yields

$$\langle i_\ell, Y_c v_s \rangle = \langle i_L, Y_c v_L \rangle + \langle i_C, Y_c v_C \rangle \quad (22)$$

where we have used the fact that, because of (5), $\langle Y_c v_R, i_R \rangle = 0$ for LTI resistors. Condition (19) is obtained directly from Definition 7.

Now,

$$\begin{aligned} \langle i_L, Y_c v_L \rangle &= \langle \nabla H_L, Y_c \dot{\phi}_L \rangle \\ &= - \left\langle \nabla^2 H_L v_L, \left(\frac{1}{p} Y_c \right) v_L \right\rangle, \end{aligned}$$

where the first identity follows from the relations (16) and the second uses the well-known property of periodic functions $\langle f, \dot{g} \rangle = - \langle \dot{f}, g \rangle$. Similar derivations with the term $\langle i_C, Y_c v_C \rangle$ yield (20).

To prove iii) we use (18), the basic relations for LTI inductors and capacitors

$$\hat{I}_{C_q}[k] = jk\omega_0 C_q \hat{V}_{C_q}[k], \quad \hat{V}_{L_q}[k] = jk\omega_0 L_q \hat{I}_{L_q}[k],$$

and the fact that Y_c satisfies (5). ■

The following remarks are in order.

- R1 Condition (19) indicates that the PF will be improved if and only if the overall weighted power (supplied plus stored) is negative.
- R2 From (20) (or replacing (21) in (19)) we see that PF improvement is equivalent to average power equalization between inductors and capacitor—notice the minus signs—with the gap being determined by the weighted supplied power.
- R3 The results of Proposition 5 in [5] are a particular case of Proposition 8 taking $Y_c = C_c p$. In particular, the aforementioned equalization interpretation of PF improvement extends the one given in [5].
- R4 In Proposition 8 it has been assumed that the resistors are linear, which is not required in [5]. This stems from the fact that we have been unable to show that $\langle i_R, Y_c v_R \rangle = 0$, for nonlinear resistors and general LTI filters. In the case of $Y_c = Cp$ (or $Y_c = \frac{1}{Lp}$) this property is easily established, invoking the periodicity assumption, and integrating by parts.

IV. IDEAL POWER-FACTOR COMPENSATION

We have shown above that $PF > PF_u$ if and only if (13), which we repeat here for ease of reference,

$$\|i_c\|^2 + 2\langle i_c, i_\ell \rangle < 0, \quad (23)$$

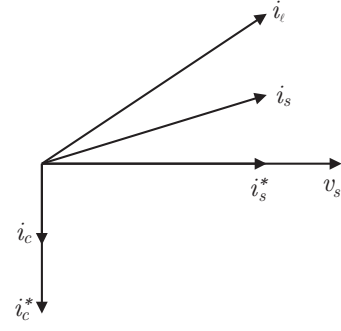


Fig. 3: Geometric interpretation of power-factor compensation

holds. On the other hand, as explained in Subsection II-A, $PF = 1$ if and only if v_s and i_s are collinear. From the facts that v_s and i_s^* are collinear and Y_c is lossless, we have

$$\langle i_s^*, i_c^* \rangle = 0,$$

where $(\cdot)^*$ notation is used to underscore that we are dealing with the optimal solution. Now, replacing $i_s^* = i_c^* + i_\ell$ in the condition above we obtain the condition for optimality

$$\|i_c^*\|^2 + \langle i_c^*, i_\ell \rangle = 0. \quad (24)$$

Comparing (24) with (23) we notice that there is a gap between PF improvement and optimality. Referring to Fig. 3 we have a (rather obvious) geometric interpretation of this gap. While the PF improvement condition (23) ensures that $\|i_s\| < \|i_\ell\|$, the optimality condition (24) places i_s^* orthogonal to i_c^* .

Current research is under way to further explore this observation for compensator synthesis.

V. APPLICATION

Consider the two linear loads in Figure 4. The parameters of both loads are chosen such that both loads cannot be distinguished in terms of Fryze's power components whenever they are supplied with the same non-sinusoidal voltage source,

$$u_s(t) = 100\sqrt{2}(\sin \omega_1 t + \sin 3\omega_1 t)V, \quad \omega_1 = 1 \text{ rad/s}.$$

Both loads have the same active power $P = 10 \text{ kW}$, apparent power $S = 14.1 \text{ kVA}$, and, consequently, the same power factor $PF = 0.71$, see [3].

In [3] it has been showed that in Fryze's framework using the parallel LC filter of Figure 5, the PF of load Y_1 is improved to $PF = 1$ with $L_c = \frac{4}{3} H$ and $C_c = \frac{1}{4} F$. However, in this framework the highest possible PF that can be obtained by a shunt reactive compensator for the load Y_2 is $PF = 0.78$, with $L_c = \frac{20}{9} H$ and $C_c = \frac{3}{20} F$. Fryze's power theory does not explain why only one of the loads considered below can be improved to unitary power-factor. In order to get a complete result, we now use our framework as presented in the previous sections. From (24) we have a condition for the compensator to yield a unitary PF

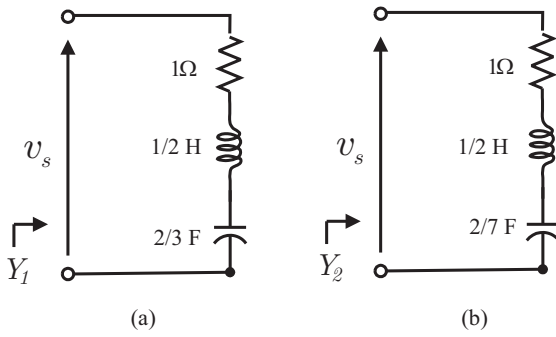


Fig. 4: Two loads that cannot be distinguished in terms of Fryze's active and reactive powers.

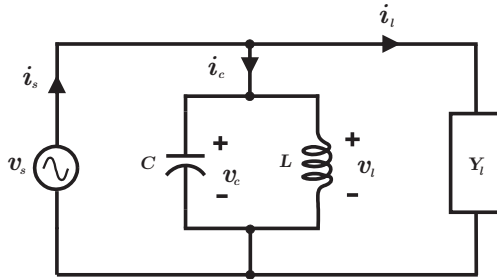


Fig. 5: Parallel LC compensation of the loads, $Y_\ell = Y_1$ or $Y_\ell = Y_2$.

given by $\|i_c^*\|^2 + \langle i_c^*, i_\ell \rangle = 0$. Then, we define the function $f(C_c, L_c) = \|i_c^*\|^2 + \langle i_c^*, i_\ell \rangle$ where the optimum values of the parameters (C_c^*, L_c^*) of Y_c , in the sense that v_s and i_s^* are collinear, must satisfy:

$$f(C_c, L_c) = \sum_{k=1,3} \frac{1 - (k\omega_0)^2 L_c C_c}{\omega_0^2 L_c} \left\{ L |\hat{I}_L(k)|^2 - C |\hat{V}(k)|^2 \right\} + \sum_{k=1,3} \frac{1 - (k\omega_0)^2 L_c C_c}{k\omega_0^2 L_c} |\hat{V}_s(k)|^2 = 0 \quad (25)$$

A solution of (25) is $C_c^* = \frac{3}{20} F$ and $L_c^* = \frac{20}{9} H$. Now consider a subset $\gamma = (0, 0.5] \times (0, 10] \subset \mathbb{R}_+^2$. Figure 6 shows the plot of level sets of $-f(C_c, L_c)$. All $(C_c, L_c) \in \beta = \{(C_c, L_c) \in \gamma | f(C_c, L_c) = 0\}$ achieve v_s and i_s^* to be collinear. However, in order to achieve unity power factor, from (8) we know that the constraint $\langle i_\ell, v_s \rangle - \|i_s^*\| \cdot \|v_s\| = 0$ must be satisfied for the compensator. The graph of this constraint is showed in Figure 7 for $(C_c, L_c) \in \gamma$. It is clear from this figure that no element of β fulfills this constraint. Hence, it is not possible for Y_2 to render a unity power factor in the configuration of Figure 5.

VI. CONCLUSIONS

In this paper, extensions to the analysis of power factor compensation of nonsinusoidal networks based on cyclo-dissipativity were presented. First, power factor compensation and a new cyclo-dissipativity condition were shown to be equivalent. Second, we have proved that the power factor is improved if and only if a certain equalization condition between the weighted powers of compensator and

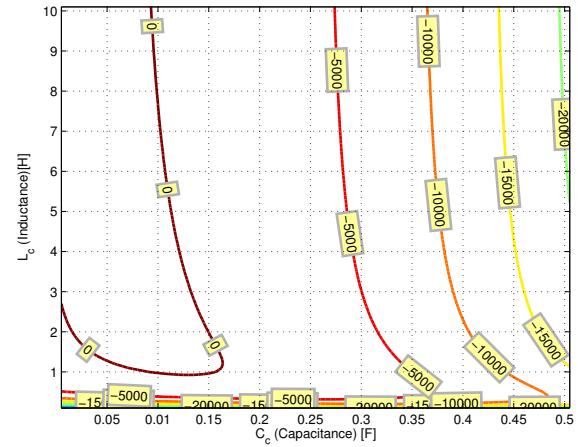


Fig. 6: The plot of level set of $-f(C_c, L_c) = 0$

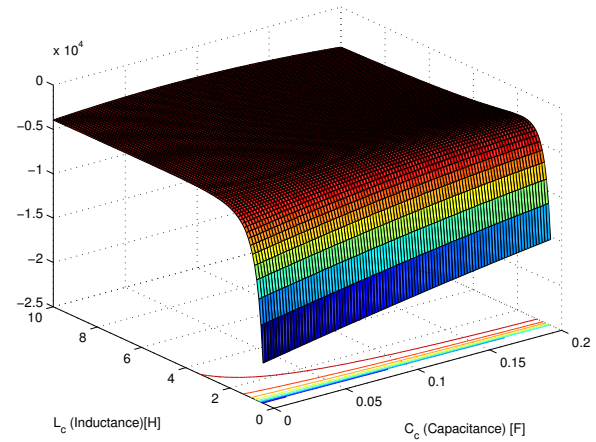


Fig. 7: Plot of the constraint $\|i_s^*\| \cdot \|v_s\| - \langle i_\ell, v_s \rangle = 0$

load is ensured. For this purpose, general lossless linear filters as compensators were considered. The gap between the ideal compensator, i.e. the one that achieves unitary power factor, and one which holds the aforementioned equalization condition was described. Finally, through an example we have illustrated that the results reported here can be used for the formulation of a problem of optimization of the compensator.

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